26.09.2023 - Practice lesson 2

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OPTIMIZATION PROBLEMS ON RIVER QUALITY

Exercise 1 – Maximum admissible load

REQUIRED FILE: 2023.09.26_Practice2.xslx | sheets "ex1_calibration", "ex1_maxDISCH"

The Burgy-ham industry is building a new production unit in order to increase its production. Its engineers are designing the treatment plant for the biodegradable discharge: in order to define the required performance of the treatment plant, they are trying to quantify the maximum concentration of BOD which allows to be compliant with the legislation. According to the most recent law, the minimum concentration of dissolved oxygen in a river must be 3 mg/l. The Blue Creek, where the Burgy-ham industry is located, is characterized by a constant velocity of 3.25 km/h. Moreover, the environmental agency observed the following data on the creek.

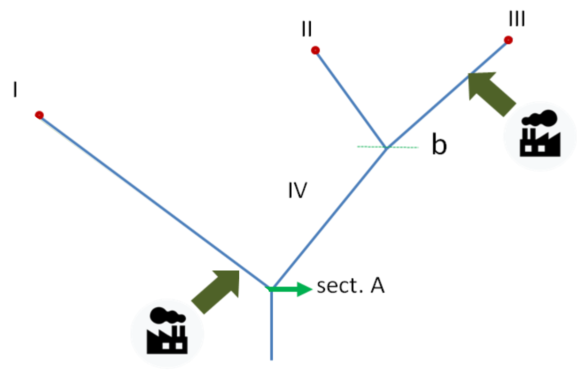
Km	BOD	DO
0	106	7.5
2	90	3.5
4	85	3.5
8	79	4.3
14	70	6
20	60	6.5
25	55	7
32	46	8
40	38	9
60	20	9.5

The industry and its discharge pipe are located at the beginning of the 10th km. After formulating the model of the torrent (calibrating the parameters k_1, k_2 and DO_sat by minimizing the mean squared error between DO estimates and observations), define the maximum concentration of BOD the Burgy-ham industry can discharge in the Blue Creek, knowing the measurement point of DO is located at the end of the 10 km. How does the result change, if the law requirements are applied at the beginning of the 30th km downstream?

Exercise 2 – Maximum admissible load

REQUIRED FILE: 2023.09.26_Practice2.xslx | sheet "ex2_FW", "ex2_SS", "ex2_Annual_avg"

The following figure shows a sketch of the central part of the Aust river.



In the following table, the main characteristics of each branch are listed.

BRANCH	LENGTH	FLOW RATE	SECTION	BOD (0)	DO (0)
I	10 km	9 m^3/s	11 m^2	2 mg_{BOD}/l	9 mg_{DO}/l
II	5 km	6m^3/s	8 m^2	2 mg_{BOD}/l	7 mg_{DO}/l
III	4 km	9.5m^3/s	15 m^2	2 mg_{BOD}/l	9 mg_{DO}/l
IV	2 km	/	15 m^2	/	/

The seasonal temperatures for fall and winter, and for spring and summer, are reported in the following table.

BRANCH	TEMPERATURE Fall-Winter	TEMPERATURE Spring-Summer
I	6 ° C	21 ° C
II	5 ° C	18 ° C
III	5 ° C	19 ° C
IV	6 ° C	20 ° C

Along branch I an industrial plant is located. The overall BOD load discharged at the end of branch I is equal to 90 mg_{BOD}/l, with an initial DO equal to 4 mg_{DO}/l (the flow rate of the discharge is equal to 1 m^3/s).
The de-oxygenation constant k_1 is equal to 0.4 h^{-1} in the whole river. In contrast, the aeration constant k_2 depends on the water temperature in each branch, according to the following formula:

$$k_{2\_T} = k_{2\_20°} * \Theta_r^{(T-20)}$$

Where $k_{2\_20°}$ is equal to 0.2 h^{-1} and Θ_r is equal to 1.024.
The DO saturation concentration of oxygen in water (DO_{sat}, mg l^{-1}) is a function of the water temperature too. Elmore and Hayes (1960) derived an analytical expression for DO_{sat} as function of temperature T (for T between 5 and 25°C):

$$DO_{sat} = 14.652 - 0.41022 * T + 0.007991 * T^2 - 0.000077774 * T^3$$

At the main confluence (sect. A, in the figure) an annual average concentration of DO equal or higher than 4 mgDO/l is required.

Along the branch III, a discharge coming from an industry belonging to the agri-food sector is located at the beginning of the 4th kilometer (from the origin); its flow rate is equal to 1 m^3/s and initial DO equal to 4 mg_{DO}/l. The industry must guarantee an annual average value of DO concentration of 4 mg_{DO}/l (through oxygenation plant) in section A, but would like to know the maximum admissible BOD discharge.

Determine the maximum admissible discharge of BOD along the branch III, in order to respect the requirement in section A.

ADDITION: Now assume the industry on branch III would like to discharge 345 mg_{BOD}/l. Assuming this value of BOD discharged in branch III, determine the minimum initial DO that should be guaranteed by the industrial plant on branch I, in order to be compliant with the requirement (on the annual average DO) of 4 mg_{DO}/l in section A.

Exercise 3 – OPTIONAL - Optimal location for a given discharge

REQUIRED FILE: river.mat

A new industrial discharge (pipe) has to be placed at a certain distance from the city centre (CC, km_{cc}=0), where the measured concentration of BOD is equal to 163 mg_{BOD}/l and DO is equal to 3 mg_{DO}/l. The new discharge (pipe) is characterized by a concentration of biodegradable pollutant of 600 mg_{BOD}/l and a concentration of dissolved oxygen equal to 1 mg_{DO}/l. The flow rate of the pipe is equal to 8 m^3/s.
The parameters describing the river are reported in the following table (structure 'river.mat' file):

PARAMETERS		VALUE	UNIT
De-oxygenation constant	k_1	0.84	1/hour
Re-aeration constant	k_2	18	1/hour
DO concentration at saturation	$DO_{\_sat}$	10.65	mg_{DO}/l
River section(average)	$A_{\_riv}$	6	m^2
River velocity	$v_{\_riv\_km\_hr}$	3	km/hr

The best location will be selected on the basis of a Water Quality Indicator (WQI) and on the costs of the infrastructure. The WQI is obtained by combining the values of BOD and DO concentration. The weights are 0.2 for BOD (w_{BOD}) and 0.8 for DO (w_{DO}). The components of WQI are defined as follow (numerical data are included in the structure 'river.mat', while the equations are already written in the function 'quality_ind.m'):
For DO (ind_DO):

ind_DO definition	Condition
$ind\_DO_i = 0$	$DO_i/DO_{\_sat} <= 0.08$
$ind\_DO_i = ((DO_i/DO_{\_sat})*100)/1.5$	$0.08 <= DO_i/DO_{\_sat} < 0.3$
$ind\_DO_i = 30 + ((DO_i/DO_{\_sat})*100)/1.1$	$0.3 <= DO_i/DO_{\_sat} < 0.6$
$ind\_DO_i = 84.5455 + ((DO_i/DO_{\_sat})*100)/0.95$	$0.6 <= DO_i/DO_{\_sat} < 0.98$
$ind\_DO_i = 100$	$0.98 <= DO_i/DO_{\_sat} <= 1$
$ind\_DO_i = 100-[(((DO_i/DO_{\_sat})*100)-100)/2];$	$DO_i/DO_{\_sat} > 1$

For BOD (ind_BOD):

ind_BOD definition	Condition
$ind\_BOD_i = 0$	$BOD_i > 260$
$ind\_BOD_i = 5 - 0.0313*(BOD_i - 100)$	$100 <= BOD_i <= 260$
$ind\_BOD_i = 40 - 0.5385*(BOD_i - 35);$	$35 <= BOD_i < 100$
$ind\_BOD_i = 100 - 2.4*(BOD_i - 5);$	$5 < BOD_i < 35$
$ind\_BOD_i = 100$	$0 <= BOD_i <= 5$

The costs depend on the distance from the city center (km_i). A first contribution regards the costs due to the damage to the town (mitigation actions close to the city center), while the second contribution is due to the infrastructure.

$$cost_i = 50*(km_i^{-0.6}) + 60*(km_i)^{0.1}$$

1. Through the simulation of BOD and the DO along the torrent, determine the best position of the discharge along the river (considering as maximum distance 100 km from the city center) according to the only WQI and the overall indicator (J_{tot}) obtained by the combination of WQI and cost (the weights are w_{WQI}= 0.23, and w_{cost}=0.77): $J_{tot} = WQI_j * w_{WQI} - (cost_j/4) * w_{cost}$.

   [j is the index for each alternative, i.e., alternative locations for the discharge]
2. The municipality analyzed the territory along the torrent and declared that the most suitable area is between the 45th and the 55th km. Within this interval, which is the best location (km) according to the overall indicator (J_{tot})?

SOLUTIONS

Exercise 1 – Maximum admissible load

For this exercise the Excel Solver is needed. You can activate is following the guidelines at this link: https://www.solver.com/excel-solver-how-load-or-start-solver. It will be located under the 'Data' tab in Excel.

You can find the solutions of the first part of the exercise (model calibration) in file "2023.09.26_Practice2.xlsx", spreadsheet "ex1_calibration_SOL". The Solver is used to calibrate the model, by defining as objective to minimize the mean squared error between observed and estimated data (cell B38), and by changing the variable cells (cells H33, H34, H35). Values found: k_1=0.14, k_2=1.92, DO_{sat}=10.12. Since the optimization problem is non-linear, the result obtained can depend on the initial values assigned to the parameters.

The solution to the second part of the exercise (finding the maximum discharge allowed) is in spreadsheet "ex1_maxDISCH_SOL". The Solver is used to find the maximum admissible discharge, i.e., it maximizes the discharge at the beginning of km 10 (cell D26), by changing its value, subject to the constraint that DO at the end of km 10 (equivalent to beginning of km 11, i.e., cell E27, is larger or equal than 3 mg/l). Maximum u_{BOD} found = 48.79 mg/l.

The additional question, i.e., finding the maximum discharge at km 10, with the DO requirement checked at km 30 is solved similarly: in the Solver the objective to maximize and variable cell will be again cell D26, while the constraint is applied to cell E46. Maximum u_{BOD} found = 150,52 mg/l. Hint: look at DO values along the river and comment whether there are unusual behaviors.

Exercise 2 – Maximum admissible load

You can find the solutions of this e in file "2023.09.26_Practice2.xlsx", spreadsheets "ex2_FW_SOL", "ex2_SS_SOL", and "ex_2_Annual_avg_SOL". The average concentration of BOD and DO in section A on the six months period are found (cells G97 and G98 of the spreadsheet relative fall-winter and spring-summer, respectively) after:

- computing k_2 and DO_{sat} for each branch (remember that they depend on the average temperature of the considered season)
- evaluating changes in BOD and DO concentration via discretized Eulerian models for branch I, discharge I, branch II, branch III (before and after the confluence with discharge III), discharge III, and branch IV;
- mass balances at the different confluences, i.e., confluence between branch I and discharge I, confluence between branch III and discharge III, confluence between branch II and branch III, and ultimately confluences in section A.

The maximum admissible discharge of BOD along the branch III, in order to respect the DO requirement in section A is found using the Solver (it is not possible to use the solver across different sheets, for this reason we report all the cells that we will use in the Solver in the sheet "ex_2_Annual_avg_SOL") to maximize cell B7, by changing its value, under the constraint that cell M15 is >= 4. Maximum admissible BOD discharge on branch III found = 342,84 mg/l.

ADDITION: the additional question is solved by setting a value of cell B7 (sheet "ex_2_Annual_avg_SOL") to 345 (mg/l) and solving the optimization problem where the objective is to minimize the DO of discharge I (cell B3 of the sheet "ex_2_Annual_avg_SOL"), while changing its value, subject to the same constraint we used before for the DO in section A (cell M1 of the sheet "ex_2_Annual_avg_SOL"). Minimum admissible DO discharge on discharge I found = 4.65 mg/l.

Exercise 3 – OPTIONAL - Optimal location for a given discharge

The solutions of the exercise are found in folder "2023.09.26_Practice2_Ex3_Matlab" and commented in the code.
Run the main file "Practice2_Exercise3_MAIN.m" to solve the exercise.