OPTIMAL FLOW ALLOCATION IN MULTIRESERVOIR NETWORKS: THE ZAMBESI SYSTEM

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ABSTRACT: Water supply planning in a multireservoir system over a fixed time horizon is a complex problem for which different approaches have been proposed. For instance, optimal flow allocation problems can be represented in network terms, by reproducing a "static" network for each time step in the time horizon and by introducing "temporal arcs" that connect the network at different time instants, thus replicating the effect of storage devices. Further expedients allow to consider evaporation losses and transport delays. Till very recently, the application of this method to real cases has been often hindered by the computational burden. In the last ten years, however, a number of different algorithms that allow to solve networks with tens of reservoirs over hundreds of time periods were proposed. The method has been applied to the case of the Zambesi river system in order to evaluate the effects of different management and planning options. Three different solution algorithms were tested

INTRODUCTION

The management of a multireservoir system is a complex problem for which different approaches have been proposed, without finding a definite solution, even in the simplest case of a single objective that can be expressed in formal terms.

From a conceptual point of view, a first classification of existing methods is between open-loop and closed-loop management schemes. In the first case, all the information (i.e., all the inflows) is assumed to be *a-priori* known, so that the optimal solution can be computed for the whole time horizon. In the second, the decision is taken at each time instant, only on the basis of the information currently available (for instance, reservoir storages or inflow forecasts).

It is evident that only the second scheme provides a solution that can be directly implemented for the management of existing systems, since the future inflow values are always unknown. However, the results of open-loop methods can provide useful insights into the optimal management policy and give an upper bound to the feasible performances of the system which is extremely valuable in the planning actvity, when different development projects have to be compared. This is particularly true, if the optimal solution can be determined for a wide variety of flow conditions, which is almost equivalent to say, for a long period of real inflows.

Till very recently this approach has been severely limited by the computational burden, while in the last ten years a number of different algorithms that allow to solve networks with tens of reservoirs over hundreds of time periods were proposed (see for instance Kuzcera, Cao,). In this work, the analysis of various development projects in the Zambesi river system is carried out by formulating and solving as many management problems.The paper is organized as follows: a category of multireservoir system management problems is formulated in the next section as a purely network optimization procedure the caracteristics of the Zambesi basin and of the asosociated management problems are described in the two following sections; the fifth section shrtly presents three

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solution algorithms; finally, in the last section different development alternatives are analysed.

PROBLEM FORMULATION

A river system can almost always be represented as a network, i.e., a finite set of nodes connected by a finite set of unidirectional arcs (Hu 1969). An arc is characterized by the utility function U_a that depends on the value f_a of the flow passing through it, and by the upper and lower flow bounds u_a and l_a, respectively, that may reflect not only physical limits, but also legal constraints (e.g., the minimum flow required for sanitary conditions).

The nodes can be classified into three categories, on the basis of their external input q_n: *sources*, if the external input is positive; *sinks*, if it is negative; and *transitions* node, if it is null. The last category may in turn be subdivided into *junctions* and *diversions*, depending on the number of entering and exiting arcs. A network with many sources and sinks has always an equivalent network with a single source and a single sink. The equivalent network can be obtained from the original one by the introduction of a master-source (master-sink), connected to all the sources (sinks) by arcs with lower and upper bounds equal to the external inflows (outflows) and null utility. For this reason, in the following only networks with a single source and a single sink will be considered, without loss of generality.

If the utility of every arc is proportional to the flow through it and all other constraints are linear, the network is linear and the problem of optimizing the overall utility can be represented as a standard linear programming problem. This is quite straightforward when no reservoir exist, i.e., when the problem is static.

Troubles arise when the problem is non-linear and dynamic, as in most real cases (Kennington and Helgason 1980; Kuczera 1989; Guariso et al. 1993).

As far as non-linearity is concerned, a linear network formulation can still be obtained when all the non-linear utility functions can be well approximated by piecewise linear, concave and separable functions (see Fig. 1). It must be underlined that this is not a relevant limitation in many practical applications, since such an approximation can be often applied to the functions given by economic analysis (Kennington and Helgason 1980; Guariso et al. 1993).

When the function U_a satisfies the above conditions, the marginal utility P_a is piecewise constant (see Fig. 1) and one can introduce an arc for each flow interval $\left[f_{a,i}, f_{a,i+1}\right]$ in which the utility is constant, with a capacity equal to $f_{a,i+1} - f_{a,i}$. The resulting sheaf of arcs is equivalent to the original arc, while the form of the function guarantees that the optimal solution will allocate water to fill the capacity of the current arc, starting from the first arc (with the highest marginal utility), before turning to the next one. In this way the non-linear problem can be reformulated in linear network terms, at expenses of a (possibly significant) growth of the number of arcs.

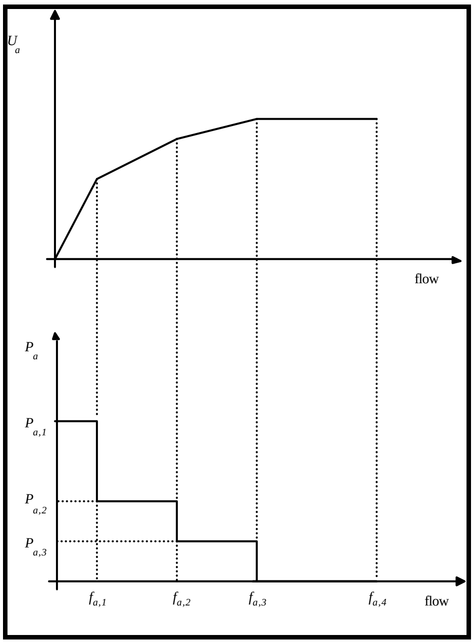

Fig. 1. Utility and Marginal Utility Functions

To consider the dynamic aspects of the problem (i.e., the fact that water can be stored in the reservoirs for a subsequent use and that the utility/cost functions and/or the constraints may vary in time), the network can be further extended. (Guariso et al. 1993; Kuczera 1989). Let us consider a problem defined over T time steps (planning horizon), each characterised by the external inflow F^t (remember that we are considering only networks with a single source and a single sink), which is supposed to be known and entirely processed (routed to destination or stored) in the same time step. A new class of nodes, the *reservoirs*, is introduced. A reservoir node b must satisfy the following continuity equation:

$$w_b^{t+1} = w_b^t + \sum_{a \in A(b)} f_a^t \tag{1}$$

where $A(b)$ is the set of arcs connected to node b, and f_a is positive when a enters b, and negative in the opposite case. Initial and final storages are considered as additional constraints and a utility (cost) function, associated to the storage values, is introduced for each reservoir. Obviously, these functions must be piece-wise linear, concave and separable.

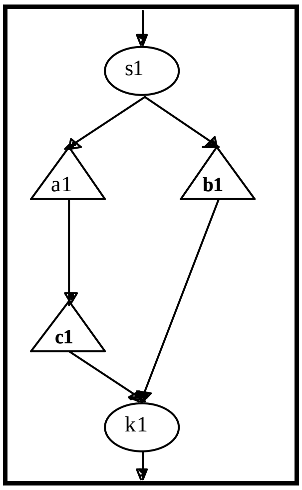

Fig. 2. A Network with Storages

Reservoirs can be introduced in the network formulation by adopting the following procedure. First, a copy of the original network (see, e.g., Fig. 2) is created for each time step t. The nodes that represent the same reservoir at consecutive time steps are connected by new (*temporal*) arcs. Then, a hyper-source and a hyper-sink are added, in such a way that the total inflow in the interval T is correctly distributed among the different time steps. This is again achieved simply by connecting the hyper-source to each source through arcs with capacity equal to the corresponding inflow and with null utility. The connection between sinks and hyper-sink must provide, instead, a free distribution of flow among the different time steps. This is made possible by connecting sinks and hyper-sink through arcs with very large (infinite) capacity and with null utility.

Initial (final) storage in the reservoirs are taken into account by connecting the hyper-source (hyper-sink) to the reservoirs in the network associated with the first (last) time step through arcs with lower and upper bounds exactly equal to the initial (final) conditions and with null utility. This flow volume must be taken into account in the hyper-source and hyper-sink flow balances. The resulting supergraph is represented in Fig. 3.

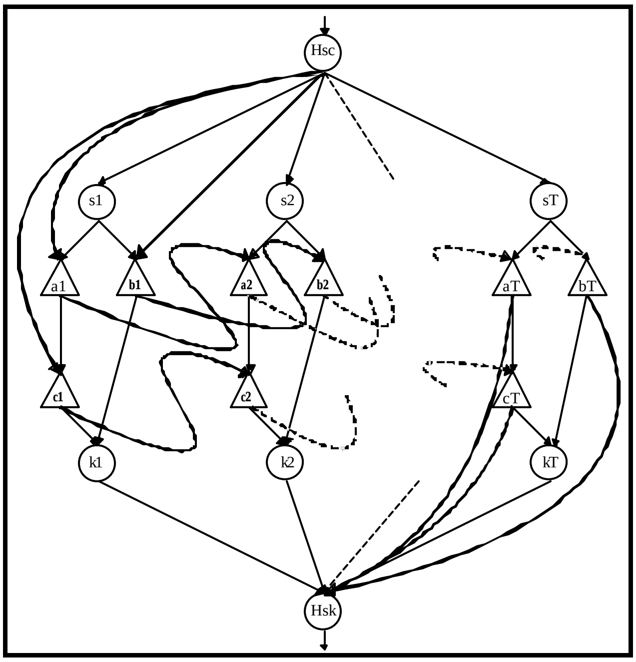

Fig. 3. Supergraph with Temporal Connections

Some generalizations can be introduced in order to deal with situations often encountered in practice.

The first case is when losses (e.g., evaporation, infiltration) from the reservoirs are not negligible. If the amount of water e_b^t that is lost from reservoir b at time t is fixed, it can be easily taken into account by connecting the reservoir with the hyper-sink through an arc with upper and lower bounds exactly equal to e_b^t and with null utility. In this way, part of the stored water is switched directly to the hyper-sink and cannot be allocated at a later time. For the sake of clarity, one can also introduce an additional node, say *waster*, between the reservoirs and the hyper-sink to collect all the lost flows.

Clearly, the assumption of fixed e_b^t may be sometimes critical. Evaporation, for example, is proportional to the reservoir surface, that may vary significantly with storage. In these cases, one could apply an iterative approach to estimate evaporation, however great care must be taken, because convergence to the correct solution is not guaranteed (Kuczera 1989).

Concentrated or distributed losses on arcs can also be taken into consideration by the introduction of "virtual" buffers along the desired arc, each of which with lower and upper bounds equal to zero (i.e., they cannot store anything), but with an associated loss. The same artifice could be useful whenever distributed utility functions along arcs are present.

Finally, when the travel time of the flow in one arc is larger than one time step, but fixed, the starting node of the arc can be connected to the ending node belonging to the network duplicate a number of time steps ahead equal to the travel time. The hyper-source and hyper-sink flow balances must be modified accordingly. Once more, this is just an approximation since in general, travel times depend on the flow istelf. Furthermore, care must be taken when connecting arcs that would have the destination node outside the interval *T.* They must in fact be reconnected with the hyper-source (incoming arcs) or to the hyper-sink (outgoing arcs) with lower and upper bounds set to fixed values. The resulting supergraph appears now as in Fig. 4.

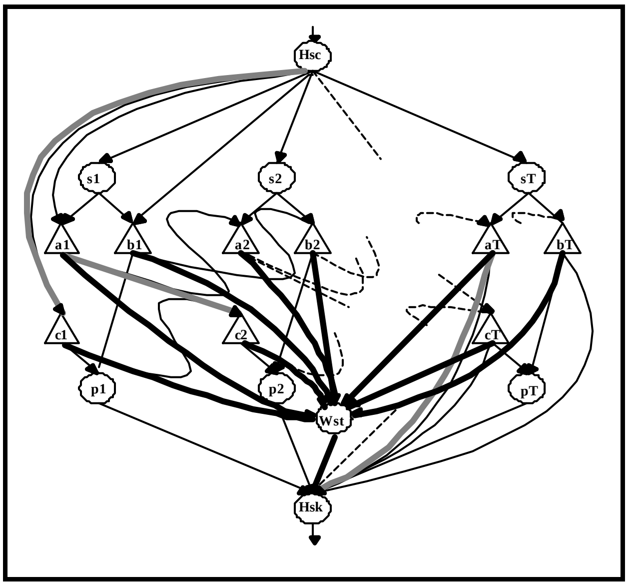

Fig. 4. Modified Supergraph

The non-linear dynamic problem of water assignement in a multireservoir system is reconducted in this way to a classic minimum cost flow allocation problem of the form:

$$\max_{\{f_a^t, w_b^t\}} \sum_t \left[\sum_a U_a^t\left(f_a^t\right) + \sum_b U_b^t\left(w_b^t\right) \right] \tag{2a}$$

subject to:

$$\sum_{a \in A(n)} i_a f_a^t = q_n^t \qquad \text{(flow balance)} \tag{2b}$$

$$l_a^t \le f_a^t \le u_a^t \qquad \text{(capacity constraints)} \tag{2c}$$

$$w_b^{t+1} = w_b^t + \sum_{a \in A(b)} i_a f_a^t \qquad \text{(continuity equation)} \tag{2d}$$

$$l_b^t \le w_b^t \le u_b^t \qquad \text{(storage constraints)} \tag{2e}$$

$$w_b^0 = \underline{w}_b \qquad \text{(initial condition)} \tag{2f}$$

$$w_b^T = \overline{w}_b \qquad \text{(final condition)} \tag{2g}$$

$$f_a^t \ge 0\,; \quad w_b^t \ge 0 \qquad \text{(non negativity cond.)} \tag{2h}$$

where, for each time step t: U_a^t, U_b^t are the utility functions of arc a and reservoir b, respectively; f_a^t is the flow in arc a; w_b^t is the storage in reservoir b; q_n^t is the external inflow (outflow) in node n; l_a^t, u_a^t, l_b^t and u_b^t are lower and upper bounds on arc a and reservoir b, respectively.

THE ZAMBESI RIVER SYSTEM

The Zambesi, with a length of about 2,500 km and a total catchment of about 1,300,000 km^2, is the largest of the African rivers flowing into the Indian ocean (see Fig. 5). Its basin is shared by eight countries: Angola, Botswana, Malawi, Mozambique, Namibia, Tanzania, Zambia, Zimbabwe. For many of them, the river represents the main water resource.

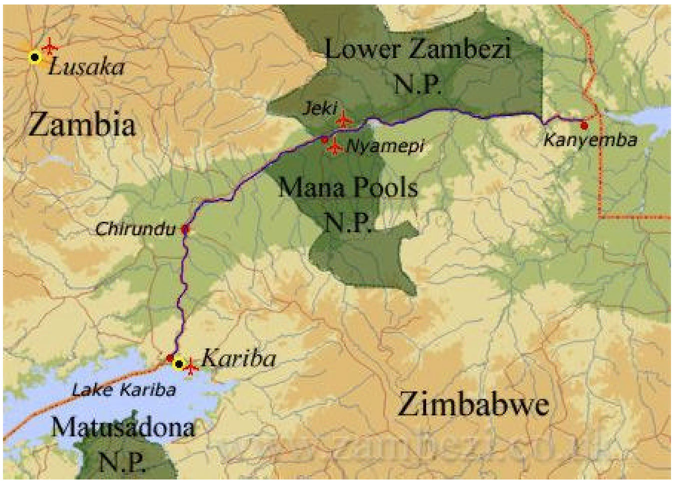

Fig. 5. Map Showing the Zambezi River Basin

Although Zambesi river has a great development potential, nowadays the main water use is limited to hydroelectric power production. For this purpose three large hydroelectric schemes were built: Kariba (Zambia/Zimbabwe) and Cabora Bassa (Mozambique) on the Zambesi itself and Kafue Gorge (Zambia) on the Kafue, one of Zambesi main tributaries. Table 1 shows the main features of the man-made reservoirs and of the power plants.

Table 1: Features of the Main Impoundments in the Zambesi Basin

	Itezhi tezhi	Kafue Gorge	Kariba		Cabora Bassa
spillway capacity (m^3/s)	4200	4250	9500		13950
live storage (m^3 x 10^6)	5 (1006-1029.5m)	0.7 (972-976.6m)	44 (475.5-488.5m)		60 (295-330m)
dead storage (m^3 x 10^6)	-- (<1006m)	-- (<972m)	116 (<475.5m)		12.5 (<295m)
			North Bank	South Bank	
number of sets	--	6	4	6	5
routed output per set (MW)	--	150	150	111	415
maximum turbine discharge (m^3/s)	--	42	200	140	452
max head (m)	--	397	108	110	128

The main characteristics of the corresponding drainage basins are briefly described in the following.

Kafue basin (from the Copperbelt region to the confluence with the Zambesi)

From the hydrologic point of view, this sub-basin can be split into three sections.

The first section covers the region between the source in the Copperbelt and the Itezhitezhi reservoir. Most of the inflows to the Kafue come from this region, where the rainy season lasts from December to March. The Itezhitezhi reservoir was built with the sole purpose of providing an additional storage capacity to the Kafue Gorge power plant, because of the limited capacity of the Kafue reservoir. A problem in the management of the scheme is caused by the large travel time between Itezhitezhi and Kafue Gorge, that, in average flow conditions, is of about two months (Pinay 1988). A record of monthly flowrates is available at Itezhitezhi starting from October 1905 (Shawinigan-Lavalin and Hidrotecnica Portuguesa 1990a).

The second section covers the region between Itezhitezhi and Kafue Gorge, that is essentially formed by the swamps of the so called Kafue Flats. The Kafue Gorge scheme was built in this point to exploit the great available head of about 400 m. However, due to the closeness of the Kafue Flats, the active storage of the reservoir is rather limited. Flow records have been kept at Kasaka, just upstream of the Kafue Gorge, also starting from October 1905 (Shawinigan-Lavalin and Hidrotecnica Portuguesa 1990a).

Finally, the incremental inflows betwen Kafue Gorge and the third section, at the confluence with Zambesi at Chirundu, is negligible.

It must be noted that the aquatic and terrestrial ecosystems of the Kafue Flats are strictly interconnected, so that the natural annual cycle of flooding and drying is of fundamental importance for the Flats ecological equilibrium. During the drying phase, from May to September, most of the aquatic flora forages wild herbivorous animals, while during the flooding phase the vegetation rapidly grows. Furthermore the flooding phase coincides with the reproduction period of a lot of fish species; therefore, an adequate flood entails an increase of the fish habitat, with positive influence on the aquatic populations. This is of real importance, not only for the ecological aspects involved, but also from the economic point of view, since the Kafue fishery is one of the most productive in Zambia (Pinay 1988).

Zambezi basin at Kariba (from the Zambesi source on the Kalene Hills to Kariba)

This basin can be split into two sub-basins. The first covers the region between the source on the Kalene Hills and Victoria Falls. A record of the monthly flowrates at Victoria Falls, starting from October 1907, has been determined from the available hydrometric data (Institute of Hydrology, 1981; Santa Clara 1988). The situation of the second sub-basin is more complex because a great number of secondary tributaries, flowing directly into the lake, often devoid of gauging stations, make the correct estimation of the overall sub-basin contribution difficult. A synthetic flow record, generated on a monthly basis by using a rainfall-runoff model for the period from October 1924 to September 1977 (CAPC 1978, 1981) is generally believed to provide a good estimate of such contribution. Monthly flows for the period from October 1907 to September 1923 (very important for its low precipitation). were estimated based on the flows at Victoria Falls, by assuming that they are in the same ratio as the respective average rainfalls.

Lake fishery is probably the first source of protein for the riparian communities. Limited lake level fluctuations (of the order of ±2m) are beneficial both for fish productivity and for navigation on the lake.

Zambezi basin at Cabora Bassa (from Chirundu to Cabora Bassa)

The incremental flow between Chirundu and Cabora Bassa is mainly due to the Huangwa river. Unforunately, no flow records of the Huangwa and of the other minor tributaries exist, and data to develop a reliable synthetic rainfall-runoff model for this region are also missing. The only way to obtain an estimate of the inflow to Cabora Bassa is to apply the continuity equation to the reservoir itself for the period from October 1930 to September 1966, the only one for which the necessary data are available. The transport delay between Chirundu and Cabora Bassa is estimated to be about 4-6 days only.

Despite, on August 2, 1984 Portugal, Mozambique and South Africa signed an agreement stipulating that two-thirds of the hydroelectric energy produced at Cabora Bassa should be sold to South Africa, almost no energy has been produced, due to the civil war that raged in Mozambique and because of sabotage of the main transmission lines. Thus, artisanal fishery has been, in these years, the main human activity on this reservoir. As in the case of Kariba, such activity would benefit of limited lake level fluctuations.

River navigation on the Zambesi final stretch, although the necessary infrastructures are far from being completed, is another aspect that might be taken into account.

Finally, water supply is not, nowadays, the major problem for the development of agriculture in Mozambique (Shawinigan-Lavalin and Hidrotecnica Portuguesa 1990b), but may become more important in the future.

THE ZAMBESI MODEL

As already stated, Zambesi reservoirs are distributed among three different countries. However, in order to avoid the complexity arising from international relationships, we assume that the power plants operate in a cooperative way. Furthermore, in the foreseable future, the non-hydro fraction of energy production in the region will probably remain relatively low so that the plants can be assumed to operate for firm energy production.

The Zambesi river system is represented by the network in Fig. 6. The various constraints are obtained from the data reported in Tab. 1, while the evaporation losses from the reservoirs are assumed periodic with values in each month equal to the respectivemonthly averages in the optimization interval. Following the same criterion, the delay in the arc Itezhitezhi-Kafue Gorge is assumed equal to the mean travel time in the stretch (2 months).

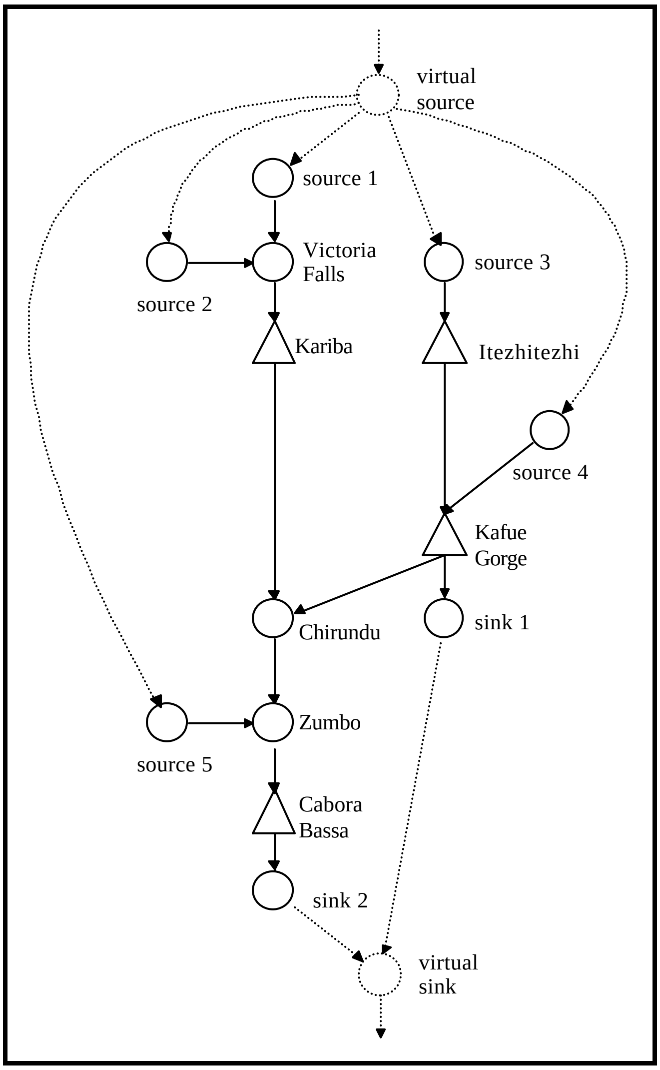

Fig. 6. Schematic of the Zambesi Network.

The energy production is assumed to be a function only of the turbine flow (dependencies on reservoir and tailrace levels and on turbine efficiency are not taken into account). The same form of utility function has been adopted for all power plants. It was derived as a 7-piece linearization of

(3)

where, for power plant i, P_i is the total installed capacity, f_i is the turbine flow and $f_i^{\max}$ is the maximum turbine discharge.

It is important to understand that Eq. (3) does not represent an attempt to estimate the actual value of hydropower, which would be very difficult, given the economies of the country of the area. It is simply a representation of the common shape of economic benefit functions (it has decreasing marginal utilities) and preserves the relative ranking between the power plants (the plant with larger installed capacity is always more profitable). Therefore, the optimal value of such a function is not of much interest in itself. On the contrary, the actual flows and power production generated by using such an objective, will be the important result.

The optimal water allocation problem was solved under different assumptions. A *reference case*, describing the present situation of the Zambesi system was first solved. Then different development alternatives that have been proposed for the system, were considered. Finally, additional constraints were introduced in order to investigate the impact of selected environmental protection strategies.

The problems were formulated with a monthly time step and solved over a temporal horizon of 144 months, from October 1930 to September 1942. The interval October 1907-September 1919, that is the dryest in the available records, was also considered for the reference case, in order to check the actual performances of the system under extreme conditions.

THE SOLUTION ALGORITHMS

Several efficient linear network algorithms are available to solve the water allocation problem formulated in the previous section. Among these, three different algorhitms were selected and thoroughly tested (Taddio and Togni 1993):

- *Netflo* (Kennington and Helgason 1980), a specialisation of the well known Dantzig's primal simplex algorhitm. to network problems
- *Relax* (Bertsekas and Tseng 1988), based on primal-dual methods.
- *Spath* (Guariso et al. 1993), based on the shortest path approach.

Several tests were carried out, based on the Zambesi network structure. Fig. 7 shows the performances of the three algorithms when the reference case is solved for an increasingly long time horizon. *Relax* and *Netflo* give similar execution times, although the increasing trend is more than linear in the case of *Netflo*, while it is approximately linear for *Relax*. The inferiority of *Spath* is apparent and was confirmed by all tests. In Fig. 8, for example, the effect of adding more reservoirs to the network is displayed. Each new reservoir is connected to the existing network by two new arcs. The time horizon is 60 time steps; therefore, for each reservoir 60 nodes and 240 arcs have to introduced in network problem. The performances of *Relax* and *Netflo* are practically coincident. Several other tests were carried out (Taddio and Togni 1993), from which *Relax* emerged as the fastest of the three algorithms. For this reason it was applied to the solution of the Zambezi case studies, presented in the following section.

Fig. 7. Execution time with increasing length of the planning horizon

Fig. 8. Computer time with increasing number of reservoirs

ANALYSIS OF THE RESULTS

The Reference Case

The reference case is characterised by the reservoir constraints and the installed capacities listed in Tab. 1, and reflects the present configuration of the Zambesi system. Further constraints are a minimum flow of 300 m^3/s in March in the Itezhitezhi-Kafue Gorge arc (to preserve Kafue Flats ecosystem) and a monthly water spill for agricultural uses of 15 m^3/s from Kafue Gorge.

In this situation, the proposed method yields a maximum firm generation powerof 3,855MW (90.9% of the installed capacity) and a mean yearly output of ??33,770 Gwh/year ***!!non è questo!!***? GWh/year. Fig. 9 shows, the firm power and the actual annual energy production during the optimization interval. It is interesting to note that the former is close to the value reported in Shawinigan-Lavalin and Hidrotecnica Portuguesa (1990b) (3500 MW), obtained by applying the HEC3 program (??) to very similar conditions.

The great lake drifts, the negative effects of which have been already discussed, are clearly shown, for instance, for Kariba in Fig. 10.

Such a variability can also be shown as in Fig. 11 where the patterns of the monthly average levels and the respective standard deviations for Cabora Bassa are reported.

It is interesting to compare the previous results with those obtained for the dry period between October 1907 and September. In fact, the analysis of this period can provide a lower bound for the Zambezi system capacity. Unfortunately, the available flow data are incomplete (the incremental flows between Kariba and Kabora Bassa are completely missing) and not fully reliable.

In order to simulate the entire network, we arbitrarily set the missing incremental flow for Cabora Bassa equal to those of the period October 1930 - September 1942 already used, even though this choice further reduces the poor data correlation. The result for this case shows the dramatic effect of the flow reduction: firm power drops to 2,588 MW (61.0% of the installed capacity) and the mean yearly output to ???Gwh/year as shown Fig. 12.

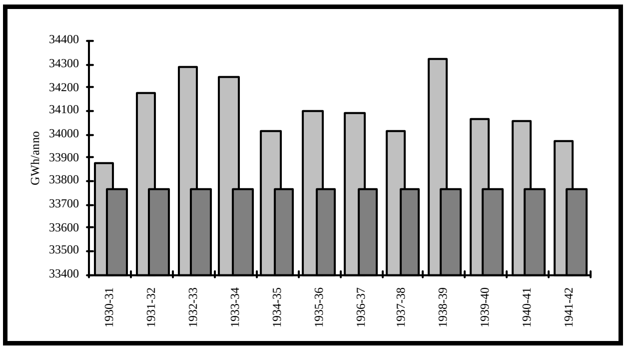

Fig. 9. Maximum Firm Power and Actual Yearly Energy Production (Reference Case).

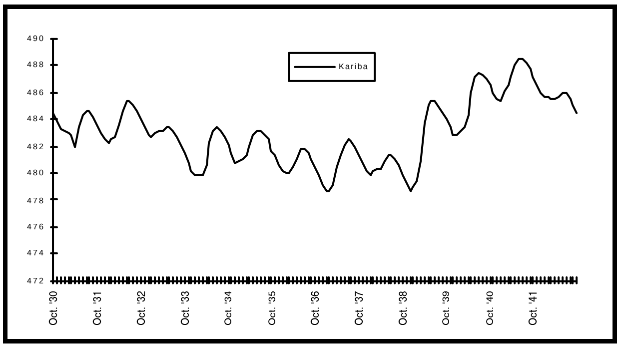

Fig. 10. Kariba Level (Reference Case)

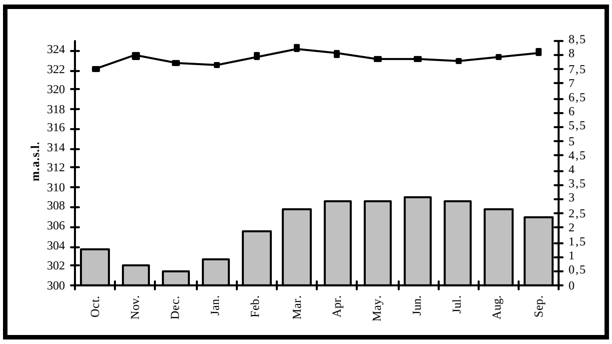

Fig. 11: Cabora Bassa average monthly level and standard deviations.

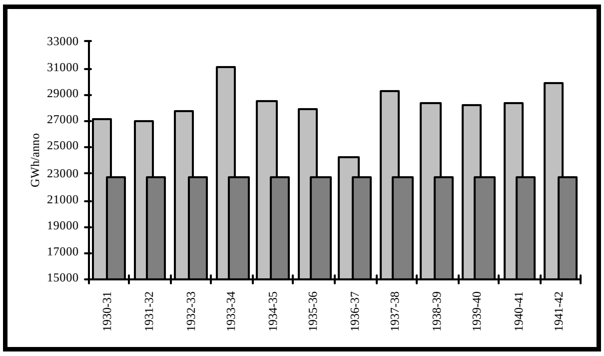

Fig. 12. Maximum Firm Power and Actual Energy Production (Reference Case, Dry Period)

This result must be carefully considered in order to avoid disastrous deficit if, as it is most likely, the future development of the region will be based only on hydroelectric production.

Power Plants Upgrade

The following modifications to existing plants are seriously being considered for implementation in the near future (Shawinigan-Lavalin and Hidrotecnica Portuguesa 1990b):

- upgrade of 1,200MW for the installed capacity at Cabora Bassa
- upgrade of 534MW at Kariba
- upgrade of 450MW for the installed capacity at Kafue Gorge, achieved through the construction of a new stage that exploits the residual head of 200m still available at the Gorge.

When all three upgrades are implemented, the resulting firm power raises to 4,355MW (67.8% of the new installed capacity). Therefore, the increase of the firm power is not as large as it could be expected and the firm to installed ratio decreases significantly. The mean yearly output raises to ??? GWh/year. Fig. 13 shows the firm power and the actual annual energy production.

The three upgrade proposals are also analyzed separately (Taddio and Togni 1993). It turns out that Cabora Bassa and, remarkably, Kariba upgrades have a very limited influence on the firm power (4,068 MW and 3,869 MW, respectively). On the contrary, the proposed upgrade of Kafue Gorge is very effective. The result obtained in this case is excellent: the firm power is 4,287 MW (91.4% of the new installed capacity) and the mean yearly output is ??? Gwh/year (see Fig. 14). In view of such result, the opportunity of increasing the live storage at Itezhitezhi of $785 \cdot 10^6$ m^3, by rising the full supply level to 1031.5 m, is also analyzed. However, no improvement in the firm energy production is achieved, because the value of the energy production in the worst month remains practically unchanged, although the mean annual production at Kafue increases.

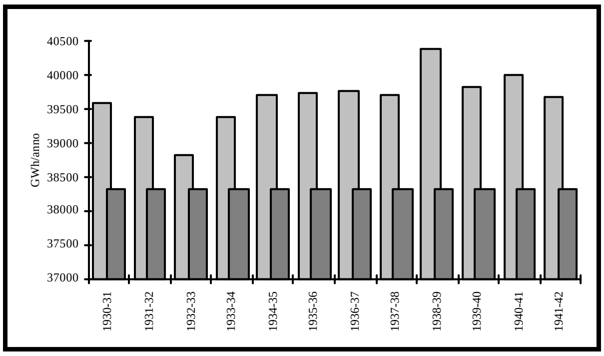

Fig. 13. Maximum Firm Power and Actual Yearly Energy Production (Upgrade of All Power Plants)

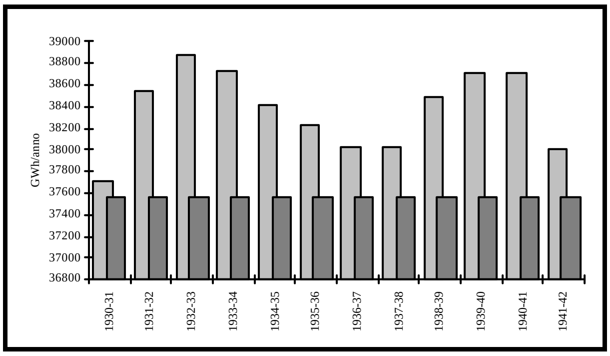

Fig. 14. Maximum Firm Power and Actual Yearly Energy Production (Kafue Gorge Upgrade)

Irrigation Development in the Kafue Region.

Among the secondary uses of the Zambezi water, the agricultural one should raise in importance in the future, As an example, the possibility of an increased water use for agricultural purposes in the Kafue region is taken into account by rising the diverted flow from 15 to 50 m^3/s. Given the characteristics of the soil in the region, this shall allow the irrigation of about 300 km^2. All other constraints are those of the reference case. In spite of its relative exiguity, the uptake may influence the optimal solution, since the Kafue Gorge scheme has, by large, the highest hydraulic head. In fact, although the firm power that is obtained (3,719 MW; 87.7 % of the installed capacity) is almost the same, a significant change of the pattern of actual energy production is apparent from Fig. 15 and the mean yearly output drops from ??? to???.

Fig. 15. Maximum Firm Power and Actual Yearly Energy Production (Irrigation Development)

Protection of Paludal Ecosystems

A greater consideration to the paludal environments in the Zambesi basin is desirable, in view of the ecological aspects involved, and also for the economic impact on fishery and tourism. Kafue Gorge, for example, with more than 5,500 tons of fish caught in 1970, was one of the most important fishery areas in Zambia.

The progressive reduction of fish production, and of the number of wild herbivores (Pinay, 1989), probably depends also on the alteration in the seasonal flooding of the Kafue Flats, due to the Zambesi regulation. Although the complex processes related to such impacts are not completely understood, it would be certainly beneficial to maintain the flooding pattern as close as possible to the natural regime.

For this reason the release constraint on the Itezhitezhi-Kafue Gorge arc has been modified by adding to the constraint on the minimum flowrate (300 m^3/s) in March two analogous constraints (150 m^3/s) for February and April as shown in Fig. 16 (Gandolfi and Salewicz 1990), in order to obtain an yearly flow pattern closer to the natural one.

Furthermore, a minimum flow of 1,800 m^3/s in March is required for the Chirundu-Cabora Bassa arc to preserve Mana Pools ecosystem. In this situation the same value of firm power as in the reference case is obtained, indicating that the new constraints are not particularly severe. Fig. 17 shows, however, that the pattern of actual energy production changes significantly and in many years the critical situations occur. Note, in fact, that the mean yearly output drops to ?? GWh/year.

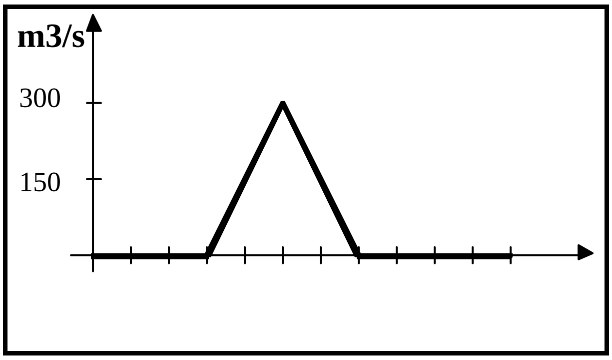

Fig. 16. Pattern of the Lower Bound on the Flow in the Itezhitezhi-Kafue Gorge Arc(Swamps Protection)

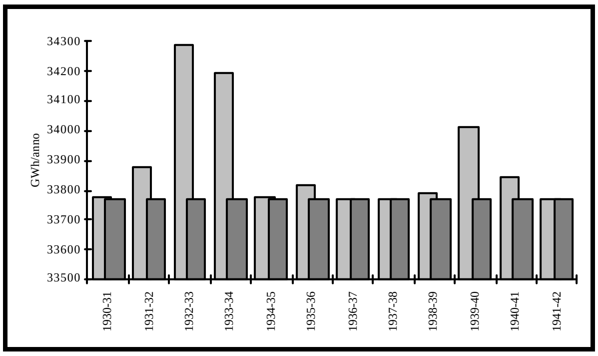

Fig. 17. Maximum Firm Power and Actual Yearly Energy Production (Swamps Protection).

Limitation of Lake Levels Fluctuations

The solution of the standard case entails large fluctuations of the lake levels planners (see Fig. 10 and Fig 11), which may damage fishery and navigation, and be undesirable for the. In order to analyse the influence of a reduction of such fluctuations, the regulation band of Kariba and Cabora Bassa reservoirs was limited to 482-486 masl and 322-326 masl, respectively. At the same time, Itezhitezhi upper level has been incremented to 1,031.5 masl, to partially compensate for the loss of active capacity.

A maximum firm power of 3,225 MW (76.0% of the installed capacity) and a mean yearly output of ??? Gwh/year were obtained. This shows that, notwithstanding the increment of Itezhitezhi upper level, narrower regulation bands on Kariba and Cabora Bassa determine a significant reduction of the power output.

The patterns of monthly levels at Kariba, with and without the constraint on fluctuations, are represented in Fig. 19.

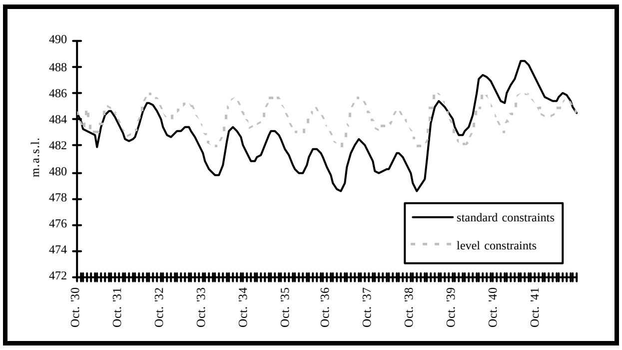

Fig. 19. Kariba Levels in the Standard Case and in the Fluctuations Reduction Case

Finally, it is worth while observing that, when the limitation of the lake level fluctuations is combined with swamp protection, the further decrease of the firm generation power is very limited (approximately 1%).

CONCLUSIONS

The applicability of three network optimization algorithms to water supply palnning in multireservoir systems was tested by solving a real problem. The best performances were given by the *Relax* algorithm (Bertsekas and Tseng 1988), which is based on primal-dual methods. Its application to the Zambesi case study allowed a quantitative analysis of several development alternatives. Fig. 20 compares the results obtained, in terms of firm energy production.

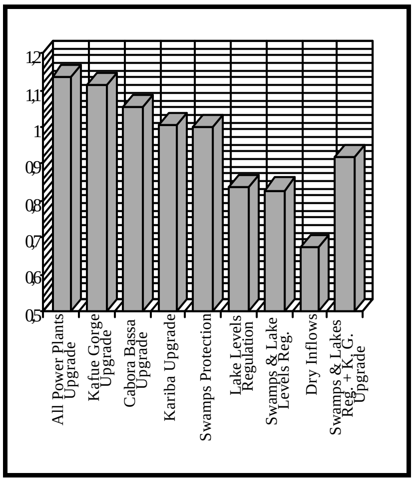

Fig. 20. Energy productions in the different cases; production in the reference case is unit

As far as the increase of the installed capacity of the system is concerned, the Kafue Gorge upgrade seems to be the most advantageous. Since this upgrade consists in a new generator stage exploiting the residual head, in cascade to the existing scheme, the great improvement that is obtained is not surprising. The Kariba upgrade and, to a lesser extent, the Cabora Bassa upgrade, on the contrary, seem to be less profitable.

From the environmental protection point of view, an increased attention to the preservation of the paludal ecosystems in Zambia and Zimbabwe through periodical flooding, can be achieved without decreasing the firm power. This result is certainly very interesting, although it must be underlined that it is certainly too optimistic, since the proposed method cannot take into account the growth in evaporation losses due to the increase in swamps surface during the flooding period. This issue should not be important in the case examined here, since the constraints on the minimum flows that were imposed in this study are not particularly severe, but it cannot be neglected when larger flows are considered.

On the contrary, the reduction of the fluctuations of the lake levels at Kariba and Cabora Bassa, by limiting the regulation band, causes a significant decrease of both the firm and the mean energy production. This loss could be partly counterbalanced by the proposed upgrade of Kafue Gorge. In such a case, in fact, the firm power is 3,536 MW, 91% of the reference case value.

Finally it must be emphasised that, in spite of the low reliability of the October 1907-September 1919 flow series, the strong reduction obtained in this extremely dry period must be taken in due consideration; especially if the development of the region will be based exclusively on hydroelectric production.

The method adopted in this study is interesting in planning studies, because it allows a fast evaluation of the more significant factors of the problem. Despite it operates in open loop, using it with a long and various inflow record, it may also gives interesting indications for the development of actual management policies.

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